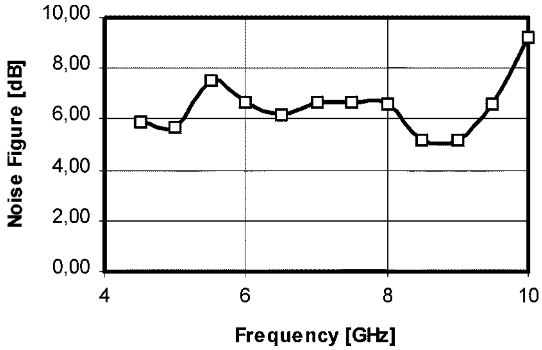

Fig. 6. Noise figure versus frequency.

The entire mixer circuit was tested with respect to the conversion gain, RF/LO isolation, noise figure, and saturation power. The dependencies of the conversion gain and the isolation between the RF and LO inputs on the frequency are shown in Fig. 5. The measurements were carried out at a transistor gate voltage of -0.6 V, a value yielding optimal characteristics at LO power of 5 dBm without using additional circuit adjustment. As one can see, we have obtained gain in the 5–7-dB range from 4.5–10 GHz with conversion down to an IF of 0.5 GHz, which agrees well with the simulation results (7–8 dB). Our experiments also show that the conversion gain is weakly sensitive to varying the LO power from 3 to 5 dBm. Setting the LO power to 3 dBm leads to a drop of gain by about 1 dB. The isolation measured between the signal and LO inputs is from 20 to 30 dB, which is 5–7 dB less than the calculated values. This is obviously due to the difference in the impedance of the transistor gates. The noise figure, shown in Fig. 6, is typically 5–7.5 dB and reaches 9 dB in the upper end of the band; it closely follows the behavior of the conversion gain. The values achieved for the noise figure are comparable with those typical for diode mixers in a wide-band mode of operation in the frequency range considered. The 1-dB compression point at the output was found to be 0 dBm.

IV. Conclusion

In this paper, we have developed and investigated a novel type of balanced active HEMT mixer in a wide-band mode of operation. The mixer is characterized by good isolation between the RF and LO ports without the use of filtering elements, conversion gain, and noise figure comparable to that of Schottky diode mixers. Input and output matching circuits are designed in order to obtain optimal conversion gain and noise figure. The conversion gain and noise figure values are typically in the 5–7- and 5–7.5-dB ranges, respectively, within the 4.5–10-GHz frequency band. The microwave part of the mixer is easy to implement which, together with the overall characteristics of the device, makes it suitable for many applications in the centimeter and millimeter ranges.

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Circular Cylindrical Waveguide Filled with Uniaxial Anisotropic Media—Electromagnetic Fields and Dyadic Green's Functions

Le-Wei Li, Shenghong Liu, Mook-Seng Leong, and Tat-Soon Yeo

***Abstract*—Electromagnetic fields in a circular cylindrical conducting waveguide filled with uniaxial anisotropic media are formulated in this paper by using Fourier transformations. These fields are obtained as a superposition of the TE (or ordinary) and TM (or extraordinary) modes satisfying, respectively, different characteristic equations. Lastly, the dyadic Green's function is derived using the Ohm–Rayleigh method and represented by vector wave functions expansion.**

I. Introduction

Over the past several decades, considerable attention has been paid to the interaction between electromagnetic waves and anisotropic materials [1]–[3]. As is well known, an anisotropic medium is characterized by its permittivity tensor $\overline{\boldsymbol{\epsilon}}$ and permeability tensor $\overline{\boldsymbol{\mu}}$ [1], of which the form depends on the kind of anisotropy.

In analysis of anisotropic media, a couple of methods have been widely applied [2]–[9]. The Fourier transform relates the physical quantities in the spatial and spectral domains [2]–[6]. As an assistant, the method of angular spectrum expansion provides a way of coordinates transformation [4], [6]. The TE/TM decomposition method was used to solve electromagnetic problems involving a certain class of boundaries and media that basically separate TE- and TM-mode fields [7]–[9]. The dyadic Green's function (DGF) technique [10] is a powerful analytic method for solving boundary-value problems. Its applications in anisotropic media have already been well explored [11], [12].

In this paper, attention is paid to the analysis of the electromagnetic fields in circular cylindrical conducting waveguides filled with electrically uniaxial anisotropic media and the DGF. In obtaining the DGF, the main tasks are to find the vector wave eigenfunctions by which the electromagnetic fields can be expanded completely and then to determine the coefficients of eigenfunctions expansion. The conventional

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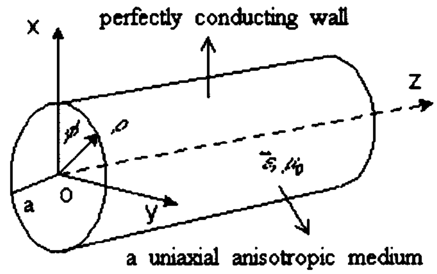

Fig. 1. Geometry of a cylindrical waveguide.

technique given in [10] cannot be directly applied and certain extension and generalization of the orthogonality relations have to be considered in the formulation. The results in this paper are reducible to that for the isotropic case, which has been obtained by Tai [10].

II. Basic Formulation of the Problem

The characteristic feature of the uniaxial media is the existence of a distinguished axis. If one of the coordinate axes is chosen to be parallel to this distinguished direction, it turns out that the parameter tensor is diagonal, but the element referring to the distinguished axis is different from the remaining two diagonal ones.

Consider a cylindrical waveguide (Fig. 1) of which coordinate axes systems are represented by $(x,\ y,\ z)$ and $(\rho,\ \phi,\ z)$. $\hat{z}$ is the direction of propagation.

The waveguide is filled with homogeneous electrically uniaxial anisotropic medium that can be characterized by the following set of constitutive relations:

$$\boldsymbol{D} = \epsilon_0 \overline{\boldsymbol{\epsilon}}_r \cdot \boldsymbol{E} \quad \boldsymbol{B} = \mu_0 \boldsymbol{H} \tag{1}$$

where ϵ_0 and μ_0 are the free-space permittivity and permeability constants, respectively. The optics axis of the uniaxial media is assumed to be oriented along the z-axis, and the other two principal axes are oriented along the two remaining coordinate axes, i.e.,

$$\overline{\boldsymbol{\epsilon}}_r = \begin{bmatrix} \epsilon_t & 0 & 0 \\ 0 & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{2}$$

From the Maxwell's equations,

$$\nabla \times \boldsymbol{E} = i\omega\mu_0 \boldsymbol{H} \quad \nabla \times \boldsymbol{H} = -i\omega\epsilon_0 \overline{\boldsymbol{\epsilon}}_r \cdot \boldsymbol{E} + \boldsymbol{J} \tag{3}$$

we have

$$\nabla \times \nabla \times \boldsymbol{E} - k_0^2 \overline{\boldsymbol{\epsilon}}_r \cdot \boldsymbol{E} = i\omega\mu_0 \cdot \boldsymbol{J} \tag{4}$$

where $k_0 = \omega\sqrt{\epsilon_0\mu_0}$, which is the free-space wavenumber. The harmonic $e^{-i\omega t}$ time dependence is assumed.

III. Fields in Source-Free Region

In the region outside the source, (4) reduces to

$$\nabla \times \nabla \times \boldsymbol{E} - k_0^2 \overline{\boldsymbol{\epsilon}}_r \cdot \boldsymbol{E} = 0. \tag{5}$$

The characteristic waves corresponding to (5) can be examined in the spectral domain using the Fourier transform

$$\boldsymbol{E}(\boldsymbol{r}) = \iiint_{-\infty}^{+\infty} \boldsymbol{E}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\, d\boldsymbol{k} \tag{6}$$

where $\boldsymbol{k} = k_x\hat{\boldsymbol{x}} + k_y\hat{\boldsymbol{y}} + k_z\hat{z}$. Substituting (6) into (5) leads

$$\iiint_{-\infty}^{+\infty} \left(k^2\overline{\boldsymbol{I}} - \boldsymbol{k}\boldsymbol{k} - k_0^2\overline{\boldsymbol{\epsilon}}_r\right) \cdot \boldsymbol{E}(\boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\, d\boldsymbol{k} = 0 \tag{7}$$

where $\overline{\boldsymbol{I}} = \hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + \hat{z}\hat{z}$ is the unit dyadic. For nontrivial solutions of (7), it is required that the determinant of matrix $(k^2\overline{\boldsymbol{I}} - \boldsymbol{k}\boldsymbol{k} - k_0^2\overline{\boldsymbol{\epsilon}}_r)$ be equal to zero. This yields the characteristic equation

$$\epsilon_t k_\rho^4 + (\epsilon_t + \epsilon_z)\left(k_z^2 - k0^2\epsilon_t\right)k_\rho^2 + \epsilon_z\left(k_z^2 - k_0^2\epsilon_t\right)^2 = 0 \tag{8}$$

with $k_\rho^2 = k_x^2 + k_y^2$ and, consequently, the eigenvalues

$$k_{\rho 1}^2 = k_0^2\epsilon_t - k_z^2 \quad k_{\rho 2}^2 = k_0^2\epsilon_z - k_z^2\frac{\epsilon_z}{\epsilon_t}. \tag{9}$$

It can be seen that $k_{\rho 1}$ is independent upon ϵ_z, while $k_{\rho 2}$ is a function of ϵ_z, which lead to the ordinary and extraordinary waves [2], respectively. The corresponding eigenvectors are given by

$$\begin{aligned} E_{1z} &= 0 \\ E_{1x}\cos(\phi_k) + E_{1y}\sin(\phi_k) &= 0 \\ \boldsymbol{E}_1(\phi_k, k_z) &= E_{1x}\hat{\boldsymbol{x}} + E_{1y}\hat{\boldsymbol{y}} \end{aligned} \tag{10a}$$

for $k_{\rho 1}$, and

$$\begin{aligned} E_{2x} &= A(k_z)\cos(\phi_k)E_{2z} \\ E_{2y} &= A(k_z)\sin(\phi_k)E_{2z} \\ \boldsymbol{E}_2(\phi_k, k_z) &= E_{2x}\hat{\boldsymbol{x}} + E_{2y}\hat{\boldsymbol{y}} + E_{2z}\hat{z} \end{aligned} \tag{10b}$$

for $k_{\rho 2}$, where $\phi_k = \tan^{-1}(k_y/k_x)$ and

$$A(k_z) = \frac{\epsilon_z k_z}{\epsilon_t\sqrt{\epsilon_z(k_0^2 - k_z^2/\epsilon_t)}}. \tag{10c}$$

Obviously, $\boldsymbol{E}_1$ is in the TE mode, which can be expressed using the vector wave function $\boldsymbol{M}$ with $\hat{z}$ as the piloting vector [10], e.g., in cylindrical coordinates

$$\boldsymbol{M}_1(n, k_z) = \left[\text{in}\frac{J_n(k_{\rho 1}\rho)}{\rho}\hat{\boldsymbol{\rho}} - \frac{\partial J_n(k_{\rho 1}\rho)}{\partial\rho}\hat{\boldsymbol{\phi}}\right]e^{i(n\phi + k_z z)}. \tag{11}$$

To have a clearer view of $\boldsymbol{E}_2$, we can express it in a cylindrical coordinate system as follows:

$$\boldsymbol{E}_2(\phi_k, k_z) = \left[A(k_z)\cos(\phi - \phi_k)\hat{\boldsymbol{\rho}} - A(k_z)\sin(\phi - \phi_k)\hat{\boldsymbol{\phi}} + \hat{z}\right] \cdot E_{2z}(\phi_k,\ k_z). \tag{12}$$

Substituting (12) into (6), we have

$$\boldsymbol{E}_2(\boldsymbol{r}) = \int_0^{2\pi} d\phi_k \int_{-\infty}^{+\infty} dk_z e^{ik_z z} e^{ik_{\rho 2}\rho\cos(\phi - \phi_k)} \boldsymbol{E}_2(\phi_k, k_z). \tag{13}$$

As shown in [4], we can assume an angular expansion for the $E_{2z}(\phi_k,\ k_z)$ component

$$E_{2z}(\phi_k,\ k_z) = \sum_{n=-\infty}^{\infty} q_2(n, k_z)e^{in\phi_k}. \tag{14}$$

Substituting (12), (14), and the well-known identity

$$e^{ik_{\rho 2}\cos(\phi - \phi_k)} = \sum_{n=-\infty}^{\infty} i^n J_n(k_{\rho 2}\rho)e^{in(\phi - \phi_k)} \tag{15}$$

into (13), after some straightforward algebraic manipulations by properly grouping the terms involved in the integrations, we end up with [4]

$$\boldsymbol{E}_2(\boldsymbol{r}) = \frac{2\pi}{k_2}\int_{-\infty}^{+\infty} dk_z \sum_{n=-\infty}^{\infty} i^n q_2(n, k_z) \cdot \left[\left(1-\frac{k_z}{k_{\rho 2}}A(k_z)\right)\boldsymbol{N}_2(n, k_z) - \frac{i}{k_2}\left(k_z - k_{\rho 2}A(k_z)\right)\boldsymbol{L}_2(n, k_z)\right]. \quad (16)$$

Vector wave functions $\boldsymbol{N}$ and $\boldsymbol{L}$, with $\hat{z}$ as the piloting vector as well, are given as

$$\boldsymbol{N}_2(n, k_z) = \frac{1}{k_2}\left[ik_z\frac{\partial J_n(k_{\rho 2}\rho)}{\partial \rho}\hat{\boldsymbol{\rho}} - nk_z\frac{J_n(k_{\rho 2}\rho)}{\rho}\hat{\boldsymbol{\phi}} + k_{\rho 2}^2 J_n(k_{\rho 2}\rho)\hat{\boldsymbol{z}}\right]e^{i(n\phi + k_z z)} \quad (17a)$$

$$\boldsymbol{L}_2(n, k_z) = \left[\frac{\partial J_n(k_{\rho 2}\rho)}{\partial \rho}\hat{\boldsymbol{\rho}} + in\frac{J_n(k_{\rho 2}\rho)}{\rho}\hat{\boldsymbol{\phi}} + ik_z J_n(k_{\rho 2}\rho)\hat{\boldsymbol{z}}\right]e^{i(n\phi + k_z z)} \quad (17b)$$

where $k_2^2 = k_{\rho 2}^2 + k_z^2$, and n can take both positive and negative integer values for a particular k_z.

From (3), we have

$$H_{2z} = \frac{1}{i\omega\mu_0}\left\{\frac{1}{\rho}\frac{\partial[\rho\boldsymbol{E}_2(\boldsymbol{r})\cdot\hat{\boldsymbol{\phi}}]}{\partial\rho} - \frac{1}{\rho}\frac{\partial[\boldsymbol{E}_2(\boldsymbol{r})\cdot\hat{\boldsymbol{\rho}}]}{\partial\phi}\right\} = 0 \quad (18)$$

which means $\boldsymbol{E}_2$ is in the TM mode. This shows that fields in a cylindrical waveguide with the configuration shown in Fig. 1 can be decomposed into TE and TM fields. To match the boundary conditions, $k_{\rho 2}$ must take $\eta = p_{nm}/a$, where p_{nm} are the roots of $J_n(x) = 0$, and $k_{\rho 1}$ must take $\lambda = q_{nm}/a$, where q_{nm} are the roots of $dJ_n(x)/dx = 0$.

In the following sections, the notations η, λ, and $\boldsymbol{M}_\eta$, $\boldsymbol{N}_\lambda$, $\boldsymbol{L}_\lambda$ will be adopted instead of $k_{\rho 1}$, $k_{\rho 2}$ and $\boldsymbol{M}_1$, $\boldsymbol{N}_2$, $\boldsymbol{L}_2$, respectively.

IV. Formulation of DGF

Equation (4) can be written in dyadic form as

$$\nabla\times\nabla\times\overline{\boldsymbol{G}}_{EJ}\left(\boldsymbol{r},\boldsymbol{r}'\right) - k_0^2\bar{\boldsymbol{\epsilon}}_r\cdot\overline{\boldsymbol{G}}_{EJ}\left(\boldsymbol{r},\boldsymbol{r}'\right) = \overline{\boldsymbol{I}}\,\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right). \quad (19)$$

Following the preceding section, the DGF $\overline{\boldsymbol{G}}_{EJ}$ and $\overline{\boldsymbol{I}}\,\delta(\boldsymbol{r}-\boldsymbol{r}')$ can also be expanded using $\boldsymbol{M}_\eta(n, k_z)$, $\boldsymbol{N}_\lambda(n, k_z)$, and $\boldsymbol{L}_\lambda(n, k_z)$ as follows:

$$\overline{\boldsymbol{I}}\,\delta\left(\boldsymbol{r}-\boldsymbol{r}'\right) = \int_{-\infty}^{+\infty} dk_z \sum_{n,m}\left[\boldsymbol{M}_\eta(n, k_z)\boldsymbol{P}_\eta(n, k_z) + \boldsymbol{N}_\lambda(n, k_z)\,\boldsymbol{Q}_\lambda(n, k_z) + \boldsymbol{L}_\lambda(n, k_z)\boldsymbol{V}_\lambda(n, k_z)\right] \quad (20)$$

$$\overline{\boldsymbol{G}}_{EJ}\left(\boldsymbol{r},\boldsymbol{r}'\right) = \int_{-\infty}^{+\infty} dk_z \sum_{n,m}\left[\boldsymbol{M}_\eta(n, k_z)\boldsymbol{a}_\eta(n, k_z) + \boldsymbol{N}_\lambda(n, k_z)\,\boldsymbol{b}_\lambda(n, k_z) + \boldsymbol{L}_\lambda(n, k_z)\boldsymbol{c}_\lambda(n, k_z)\right]. \quad (21)$$

Following the Ohm–Rayleigh procedure [10], the expansion coefficients in (20) and (20) can be determined. The resultant expressions are given by

$$\boldsymbol{P}_\eta(n, k_z) = \frac{1}{4\pi^2\eta^2 I_\eta}\boldsymbol{M}'_\eta(-n, -k_z) \quad (22a)$$

$$\boldsymbol{Q}_\lambda(n, k_z) = \frac{1}{4\pi^2\lambda^2 I_\lambda}\boldsymbol{N}'_\lambda(-n, -k_z) \quad (22b)$$

$$\boldsymbol{V}_\lambda(n, k_z) = \frac{1}{4\pi^2 k_\lambda^2 I_\lambda}\boldsymbol{L}'_\lambda(-n, -k_z) \quad (22c)$$

and

$$\boldsymbol{a}_\eta(n, k_z) = \frac{1}{4\pi^2\eta^2 I_\eta}\frac{1}{k_\eta^2 - k_{\eta 0}^2}\boldsymbol{M}_\eta(-n, -k_z) \quad (23a)$$

$$\boldsymbol{b}_\lambda(n, k_z) = \frac{1}{4\pi^2\lambda^2 I_\lambda}\frac{1}{\epsilon_z k_\lambda^2\left(k_\lambda^2 - k_{\lambda 0}^2\right)} \cdot \left[\beta_{NN}\boldsymbol{N}_\lambda(-n, -k_z) + \beta_{NL}\boldsymbol{L}_\lambda(-n, -k_z)\right] \quad (23b)$$

$$\boldsymbol{c}_\lambda(n, k_z) = \frac{1}{4\pi^2\lambda^2 I_\lambda}\frac{1}{\epsilon_z k_\lambda^2\left(k_\lambda^2 - k_{\lambda 0}^2\right)} \cdot \left[\beta_{LN}\boldsymbol{N}_\lambda(-n, -k_z) + \beta_{LL}\boldsymbol{L}_\lambda(-n, -k_z)\right] \quad (23c)$$

where

$$I_\eta = \int_0^a J_n^2(\eta\rho)\rho d\rho = \frac{1}{2}a^2\left(1-\frac{n^2}{\eta^2 a^2}\right)J_n^2(\eta a) \quad (24a)$$

$$I_\lambda = \int_0^a J_n^2(\lambda\rho)\rho d\rho = \frac{1}{2}a^2 J_{n-1}^2(\lambda a) \quad (24b)$$

$$k_\lambda^2 = \lambda^2 + k_z^2$$

$$k_{\eta 0}^2 = k_0^2\epsilon_t \quad (24c)$$

$$k_{\lambda 0}^2 = k_0^2\epsilon_t + \left(1-\frac{\epsilon_t}{\epsilon_z}\right)\lambda^2 \quad (24d)$$

$$\beta_{NN} = \epsilon_t\lambda^2 + \epsilon_z k_z^2$$

$$\beta_{NL} = (ik_z)/k_\lambda\lambda^2(\epsilon_t - \epsilon_z) \quad (24e)$$

$$\beta_{LN} = -(ik_z)/k_\lambda\lambda^2(\epsilon_t - \epsilon_z) \quad (24f)$$

$$\beta_{LL} = -(\lambda^2)(k_0^2 k_\lambda^2)\left[k_\lambda^4 - k_0^2\left(\epsilon_t k_z^2 + \epsilon_z\lambda^2\right)\right]. \quad (24g)$$

In this way, the DGF for cylindrical waveguides filled with uniaxial anisotropic medium is explicitly represented by eigenfunction expansions in terms of the cylindrical vector wave functions. In order to apply the residue theorem to (20), we must first extract the term in (20), which does not satisfy the Jordan lemma, as pointed out in [10]. To do so, we write

$$\boldsymbol{N}_\lambda(n, k_z) = \boldsymbol{N}_{\lambda t}(n, k_z) + \boldsymbol{N}_{\lambda z}(n, k_z) \quad (25a)$$

$$\boldsymbol{L}_\lambda(n, k_z) = \boldsymbol{L}_{\lambda t}(n, k_z) + \boldsymbol{L}_{\lambda z}(n, k_z) = -\frac{ik_\lambda}{k_z}\boldsymbol{N}_{\lambda t}(n, k_z) + \frac{ik_z k_\lambda}{\lambda^2}\boldsymbol{N}_{\lambda z}(n, k_z) \quad (25b)$$

and, thus, are $\boldsymbol{N}'$ and $\boldsymbol{L}'$. The subscript t and z denote their transverse vector components and their z-vector components, respectively. In terms of these functions, (20) can be rewritten in the form

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r}, \boldsymbol{r}') &= \int_{-\infty}^{+\infty} dk_z \sum_{n,m} \left\{ \frac{1}{4\pi^2\eta^2 I_\eta} \frac{1}{k_\eta^2 - k_{\eta 0}^2} \times \boldsymbol{M}_\eta(n, k_z)\, \boldsymbol{M}'_\eta(-n, -k_z) \right. \\ &\quad + \frac{1}{4\pi^2\lambda^2 I_\lambda} \frac{k_\lambda^2}{k_0^2\epsilon_z (k_\lambda^2 - k_{\lambda 0}^2)} \\ &\quad \times \left[\frac{k_0^2\epsilon_z - \lambda^2}{k_z^2} \boldsymbol{N}_{\lambda t}(n, k_z) \boldsymbol{N}'_{\lambda t}(-n, -k_z) \right. \\ &\quad + \boldsymbol{N}_{\lambda t}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, k_z) \\ &\quad + \boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda t}(-n, -k_z) + \frac{k_0^2\epsilon_t - k_z^2}{\lambda^2} \\ &\quad \left.\left. \times\ \boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, k_z) \right] \right\}. \end{aligned} \tag{26}$$

The singular term in (26) is contained in the component $\boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, -k_z)$ [10]. From (20), we note that

$$\widehat{z}\widehat{z}\,\delta(\boldsymbol{r} - \boldsymbol{r}') = \int_{-\infty}^{\infty} dk_z \sum_{n,m} \frac{1}{4\pi^2\lambda^2 I_\lambda} \frac{k_\lambda^2}{\lambda^2} \times \boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, -k_z). \tag{27}$$

Thus, (26) can be split into

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r}, \boldsymbol{r}') &= -\frac{1}{k_0^2\epsilon_z} \hat{z}\hat{z}\,\delta(\boldsymbol{r} - \boldsymbol{r}') \\ &\quad + \int_{-\infty}^{+\infty} dk_z \sum_{n,m} \left\{ \frac{\boldsymbol{M}_\eta(n, k_z)}{4\pi^2\eta^2 I_\eta} \frac{\boldsymbol{M}'_\eta(-n, -k_z)}{k_\eta^2 - k_{\eta 0}^2} \right. \\ &\quad + \frac{1}{4\pi^2\lambda^2 I_\lambda} \frac{k_\lambda^2}{k_0^2\epsilon_z (k_\lambda^2 - k_{\lambda 0}^2)} \\ &\quad \cdot \left[\frac{k_0^2\epsilon_z - \lambda^2}{k_z^2} \boldsymbol{N}_{\lambda t}(n, k_z)\boldsymbol{N}'_{\lambda t}(-n, -k_z) \right. \\ &\quad + \boldsymbol{N}_{\lambda t}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, -k_z) \\ &\quad + \boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda t}(-n, -k_z) \\ &\quad \left.\left. + \frac{\epsilon_t}{\epsilon_z} \boldsymbol{N}_{\lambda z}(n, k_z)\boldsymbol{N}'_{\lambda z}(-n, -k_z) \right] \right\}. \end{aligned} \tag{28}$$

The second integral in (28) can be evaluated using the residue theorem in the k_z-plane. The final result is given after some mathematical manipulations, for $z \gtrless z'$, by

$$\begin{aligned}\overline{\boldsymbol{G}}_{EJ}(\boldsymbol{r}, \boldsymbol{r}') &= -\frac{1}{k_0^2\epsilon_z} \widehat{z}\widehat{z}\,\delta(\boldsymbol{r} - \boldsymbol{r}') \\ &\quad + \frac{i}{4\pi} \sum_{n,m} \left\{ \frac{1}{\eta^2 k_{z\eta} I_\eta} \boldsymbol{M}_\eta(n, \pm k_{z\eta})\boldsymbol{M}'_\eta(-n, \mp k_{z\eta}) \right. \\ &\quad + \frac{k_{\lambda 0}^2}{k_0^2\epsilon_t\lambda^2 k_{z\lambda} I_\lambda} \left[\boldsymbol{N}_{\lambda t}(n, \pm k_{z\lambda}) + \frac{\epsilon_t}{\epsilon_z}\boldsymbol{N}_{\lambda z}(n, \pm k_{z\lambda}) \right] \\ &\quad \left. \left[\boldsymbol{N}'_{\lambda t}(-n, \mp k_{z\lambda}) + \frac{\epsilon_t}{\epsilon_z}\boldsymbol{N}'_{\lambda z}(-n, \mp k_{z\lambda}) \right] \right\} \end{aligned} \tag{29}$$

where z and z' are the positions of the observation and source points, respectively, measured along the $\widehat{z}$-direction, and

$$k_{z\eta}^2 = k_{\eta 0}^2 - \eta^2 \quad k_{z\lambda}^2 = k_{\lambda 0}^2 - \lambda^2. \tag{30}$$

It can be observed that (29) is reducible to the isotropic case. By letting $\epsilon_t = \epsilon_z = \epsilon$, we have exactly the same form as that obtained by Tai [10].

V. Conclusion

In this paper, electromagnetic fields in a circular cylindrical conducting waveguide filled with uniaxial anisotropic media have been analyzed. With the optics axis of the uniaxial media oriented along the $\widehat{z}$-axis of the waveguide, the electromagnetic fields can be decomposed into TE and TM modes with different propagation wavenumbers. By matching the boundary conditions, it has been found that the ordinary wave takes the form of TE modes, while the extraordinary wave takes the form of TM modes. The electric-type DGF due to the electric source has been derived using the vector wave eigenfunctions expansion. The Ohm–Rayleigh method has been applied in the formulation. The singular term has been properly extracted. As their applications, these DGFs can be used to examine both the electric and magnetic fields radiated by an arbitrary current source inside the waveguides.

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